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Fermion masses in E_6 GUT with family permutation symmetry S_3

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So far it is not clear how to extend the Standard Model to include fermion masses. Mass matrices are not univocally fixed by experimental data (masses and mixings).

The remaining arbitrariness can be reduced assuming underlying gauge and flavour symmetries.

Neutrino data give strong indication that flavour symmetry is a discrete one like S_3, A_4, S_4.

Quarks and leptons mixing and mass hierarchies are very different.
This could be a problem in unified models.

We show that in E_6 it is possible to make a distinction between neutrino and the other fermions and we study the $E_6 \times S_3$ model.

Mass hierarchies

While charged fermion mass hierarchy is large

$m_\mu/m_\tau \sim 0.06, \quad m_s/m_b \sim 0.02, \quad m_c/m_t \sim 0.005$,(E.W scale)

neutrino hierarchy is weakest $m_2/m_3 > 0.2$ (should be degenerate)

(Smirnov)

Mixing hierarchies

$$U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad V_{\text{CKM}} \approx \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 0.97 & 0.03 \\ 0.00 & 0.03 & 0.99 \end{pmatrix} \approx 1$$

H*arrison,***P***erkins and* **S***cott, Phys.Lett.B 530(2002)*

In *lepton sector* the ν_3 is maximally mixed between μ and τ flavours and $\theta_{13} = 0$ while ν_2 is equally mixed between $e, \ \mu, \ \tau$.

In *quark sector* the three mixing angles are small, the only relevant is the 1-2 Cabibbo which is smaller than the 1-2 and 2-3 angles in the lepton sector $\quad \theta^i_{\text{quark}} \ll \theta^i_{\text{leptons}}$.

Neutrino data gives us more constrains and informations on flavour symmetry than quark data where $V_{CKM} \approx 1$.

Lepton mixing and flavour symmetry (**$\mu \leftrightarrow \tau$ vs S$_3$**)

A generic matrix M_ν $\mu \leftrightarrow \tau$ invariant (S$_2$) is diagonalized by O

$$O(\theta) = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix} \sqrt{2/3} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

where the solar angle θ is not fixed by the $\mu \leftrightarrow \tau$ symmetry.

The solar angle can be obtained nicely from A_4 discret symmetry
(*Ma, Altarelli and Feruglio, Babu, Valle, Frigerio, ...*)

or by **S$_3$ discrete symmetry of permutation of three objects**
(*Harrison and Scott, Ma, Haba, Grimus, Lavoura, Koide, Kubo, Mohapatra,..*).

If S$_3 \supset$ S$_2$ and its breaking scale is $\ll M_{GUT}$ $\Rightarrow$ $\sin\theta_{sol} = 1/\sqrt{3}$
(*Caravaglios and Morisi*)

We want to select the unified gauge group G_g so that

$$G_g \times S_3$$

gives the observed fermion mass and mixing hierarchies at the electroweak scale.

To explain the different hierarchies in quarks and leptons, we assume that Yukawa are proportional to the vev of some scalars ϕ_k that break S_3, $(\overline{\psi}_i\ \psi_j)_{G_g}\ \phi_k\ \varepsilon_{S_3}{}^{ijk} \equiv \overline{\psi}_i\ \psi_j\ Y^{ij}$

Consider SO(10) $\supset$ SU(5)$\times$U$_r$(1) $\Rightarrow$ 16= 1(-5)+$\bar{5}$(3)+10(-1)

mass term	SU(5) operators	U$_r$(1)		scalar
Dirac neutrino	F ν_R	-2	$\times$	+2
up quark	T T	-2		+2

The mass terms $T\ T$ and $F\ \nu_R$ have the same charge, **thus we expect that the same scalar is at the origin of their Yukawa interaction**.

While neutrino have an approximate S$_3$ symmetry, the same symmetry is not observed in up quark sector.

It is difficult to extend the discrete permutation symmetry to all fermions in GUT (quark-lepton symmetry $\Leftrightarrow$ different hierarchies).

In SU(5) neutrino and charged leptons Yukawa couplings are distinct, however we are interested in model beyond SU(5) since:

-embedding S_3 in SU(5) requiring **only one Higgs Standard Model doubled** we obtain wrong prediction. In fact the renormalizable S_3 invariant operators give

$$\lambda_1\, T^i\, T^i\, H, \quad \lambda_2\, T^i\, T^j\, H \;\Rightarrow\; \lambda_1\, v \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \lambda_2\, v \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$T^i\, T^i\, H \;\Rightarrow\; m_u = m_c = m_t$; $T^i\, T^j\, H \;\Rightarrow$ mixing too big!

-we want to explain why Yukawa are proportional to different scalars (SU(5) $\subset G$)

-we are interested in non supersymmetric extension (in such case gauge couplings do not unify in SU(5))

S_3 approximatively exact only for neutrino (S_3 **softly** broken in S_2), while S_3 **strongly** broken in charged fermion sector.

Thus if we require **one Higgs and S_3 symmetry**, tree level Yukawa must not introduce any mass terms for charged fermions

$E_6 \supset SO(10) \times U_t(1) \supset SU(5) \times U_r(1) \times U_t(1) = SU(5) \times U_X(1)$

$\mathbf{27} = 1(4) + 10(-2) + 16(1) \quad \Rightarrow$ extra singlet x_L

contains two SM singlets that can be right-handed neutrinos

SU(5) operators	$U_r(1)$	$U_t(1)$
T T (quark up)	**-2**	**+2**
$F v_R$	-2	+2
$F x_L$	+3	+5

$H\ (-3,-5)\ \subset\ \mathbf{351'}$

The Dirac mass term $F v_R$ has different quantum numbers from all the other fermions,

A standard model doublet with $U(1)_r \times U(1)_t$ charges (-3,-5) is contained in the $\mathbf{351'}$, so **if we put the Higgs in the $\mathbf{351'}$** the renormalizable S_3 symmetric interaction

a) $\mathbf{27}_i^\alpha\ \mathbf{27}_i^\beta\ \mathbf{351'}_{\alpha\beta}$

gives **only** Dirac neutrino Yukawa interaction $\Rightarrow x^t_{Li}\ \nu_{Li}$ **diagonal**.

Operator (a), does not introduce any mass neither for quarks nor for charged leptons if we put the higgs in the $\mathbf{351'}$

thus it is possible to make difference between quarks and leptons selecting E_6 as unified gauge group $\Rightarrow$ we have studied $E_6 \times S_3$.

Neutrino and double seesaw

$27\ 27 \supset$ SU(5)	$\mathrm{U}_r(1)$	$\mathrm{U}_t(1)$	$\mathrm{U}_x(1)$
$F v_R H$	-5	-3	-8
$F x_L H\ \ (m)$	0	0	0
$v_R^t v_R\ \ (m_v)$	-10	+2	-8
$v_R^t x_L\ \ (M)$	-5	+5	0
$x_L^t x_L\ \ (m_x)$	0	+8	+8

A scalar singlet with charges (+5,-5), is contained into **351′** that could come from **78**×**351** (since the Higgs is already in the **351′**)

$$\mathrm{E}_6 \xrightarrow{\langle 78\rangle} \mathrm{SO(10)}\times\mathrm{U(1)} \xrightarrow{\langle 351\rangle} \mathrm{SU(5)}\times\mathrm{U}_x(1).$$

$$v_R^t x_L, \quad \mathbf{27}^i\ \mathbf{27}^j\ \mathbf{351}^k\ \mathbf{78} \Rightarrow M \sim \langle 351\rangle\langle 78\rangle$$

$$x_L^t x_L \text{ and } v_R^t v_R,\ \mathbf{27\ 27\ 27\ 27}\ \Rightarrow m_v,\, m_x \sim \langle 27\rangle^2$$

$$\begin{pmatrix} v_L & v_R^c & x_L \end{pmatrix}^i \begin{pmatrix} 0 & 0 & m\ \delta_{ij} \\ 0 & m_v\ \delta_{ij} & M_{ij} \\ m\ \delta_{ij} & M_{ij} & m_x\ \delta_{ij} \end{pmatrix} \begin{pmatrix} v_L \\ v_R^c \\ x_L \end{pmatrix}^j \Rightarrow\ m_v \left(\frac{m^2}{M^2}\right)_{ij}$$

where $m,\ m_v \sim m_x \ll M$. Neutrino mixing comes only from M.

We can have the following lagrangian for neutrino

$$L \; = \; \lambda \; \mathbf{27}^i \; \mathbf{27}^j \; \mathbf{351}^k \; \mathbf{78} \; + \mathbf{27}^i \; \mathbf{27}^i \; \mathbf{351}^i \; \mathbf{78}$$

$$M = \langle\psi\rangle \begin{pmatrix} \frac{\langle 351^1\rangle}{\langle\psi\rangle} + 1 & 1 & 1 \\ 1 & \frac{\langle 351^2\rangle}{\langle\psi\rangle} + 1 & 1 \\ 1 & 1 & \frac{\langle 351^3\rangle}{\langle\psi\rangle} + 1 \end{pmatrix}$$

where $\langle\psi\rangle = \lambda \;\; \left(\langle 351^1\rangle + \langle 351^2\rangle + \langle 351^3\rangle\right)$.

When $\mathbf{S}_3$ **breaks spontaneously in** $\mathbf{S}_2$: $\langle 351^1\rangle \neq \langle 351^2\rangle = \langle 351^3\rangle$.

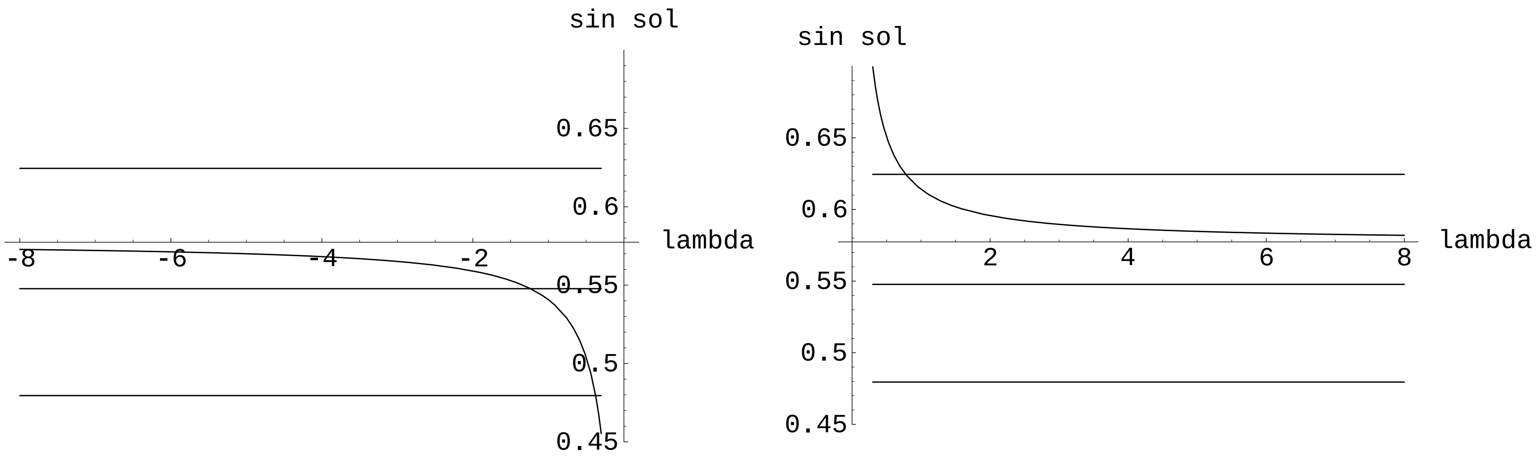
Then M is $\mu \leftrightarrow \tau$ invariant $\;\Rightarrow\;$ $\theta_{13} = 0$ and θ_{atm} maximal.

In the limit $\lambda \to \infty$ (S_3 softly broken) $\Rightarrow$ $\sin\theta_{sol} = 1/\sqrt{3}$

while $\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{\langle 351_1\rangle^2}{(\langle 351_1\rangle - \langle 351_2\rangle)^2} \approx \frac{1}{35} \;\Rightarrow\; \langle 351_1\rangle \sim 5 \; \langle 351_2\rangle$

solar angle can be fitted with λ finite.

$$\sin\theta_{sol} = \frac{1}{\sqrt{3}}\left(1+\frac{2}{9}\left(\frac{\langle 351_1\rangle - \langle 351_2\rangle}{\langle\psi\rangle}\right)\right) \equiv \frac{1}{\sqrt{3}}\left(1+f(\lambda)\right)$$



The horizontal lines are the upper and the lower limit (3σ) from the experimental central value $\sin\theta_{sol} \approx 0.547$.

In the limit $\lambda \to \infty$ we get $\Rightarrow$ $\sin\theta_{sol} = 1/\sqrt{3}$ but we can fit the solar angle with a λ of **order one** (no fine tuning).

Up quarks

SU(5) operators	$U_r(1)$	$U_t(1)$
$T^{\alpha\beta}T^{\gamma\delta}H^{\sigma}$	-5	-3

$\times$

SU(5) singlet
(+5,+3)

$$= \begin{array}{c} 27^{\alpha}\,27^{\beta}351'_{\gamma\sigma}\ \Sigma^{\gamma\sigma}_{\alpha\beta} \\ \alpha\neq\beta\neq\gamma\neq\sigma \end{array}$$

$$78^{\gamma}_{\beta}\times 78^{\rho}_{\beta}\supset\Sigma^{\gamma\sigma}_{\alpha\beta}\Leftrightarrow\ 78_{a}{}^{\gamma}_{\beta}\ ,\ 650_{s}{}^{\gamma}_{\beta}\ ,\ 2430_{s}\ ,\ 2925_{a}$$

All representations contain SU(5) singlets with (+5,+3) charges, but up mass terms cames from $\mathbf{2430}^{i}$.
$\mathbf{2430}^{i}\subset 351'\times 27\times 27$ but we have found $\Sigma\ \neq\ 351'\times 27\times 27$.
The $\mathbf{2430}^{i}$ is not proportional to $\mathbf{351}^{i}$ $\Rightarrow$ up and neutrino sectors can have inverted hierarchies.

Down quarks and charged leptons ($M_l = M_d^T$)

We need **1728**, differently from up there are two distinct operators

$$g_1\,27^{\alpha}_{\mathbf{i}}\ 27^{\beta}_{\mathbf{j}}\ 351'^{*\,\gamma\delta}\ 1728^{*\rho}_{\delta\beta\mathbf{i}}\ \varepsilon_{\alpha}\gamma\rho+g_2\ 27^{\alpha}_{\mathbf{i}}\ 27^{\beta}_{\mathbf{j}}\ 351'^{*\,\gamma\delta}\ 1728^{*\rho}_{\delta\alpha\mathbf{i}}\ \varepsilon_{\beta\gamma\rho}$$

$$\Rightarrow g_1\ d_{L\,i}\ d^{c}_{R\,j}\ h\ \ \left\langle\overline{1728}_{i}\right\rangle + g_2\ d_{L\,j}\ d^{c}_{R\,i}\ h\ \ \left\langle\overline{1728}_{i}\right\rangle$$

2430

$\mathbf{2430}^i \subset 351' \times 27 \times 27$ contains a singlet of SU(5) with charges (+5,+3). Consider , for example , the following up mass term

$$u_L^1 u_R^{1c} H \;=\; 27^{12}\, 27^{6}\, 351'_{22,26}\, \Sigma^{22,26}_{12,6}, \qquad \Sigma \neq 351'_{12,6}\, 27^{22}\, 27^{26}$$

index **27**	Q_{em}	Y	I_3^w
6*	2/3	2/3	0
12*	-2/3	-1/6	-1/2
22	0	0	0
26	0	-1/2	1/2

$$351' = \overline{27} \times \overline{27}$$

The tensor Σ must to contain simultaneously all the indices 6,12,26 otherwise it is not a standard model singlet. Observe that if the component 27^{26} take a vev, we have an extra SU(2)$_L$ doublets, but we want only one higgs particles.

Conclusion

Neutrino physics is one of the most important guide beyond SM.

In particular neutrino data give strong constrains to select the underlying flavour symmetry.

Neutrino data are well explained by discrete permutation symmetry.

Quarks and leptons hierarchies are very different so it is difficult to embed S_3 in unified gauge group.

In E_6, putting the Higgs in the $\mathbf{351}'$, the renormalizable S_3 invariant interaction give only Dirac neutrino mass term $x^t_{Li}\ \nu_{Li}$.

While we need non renormalizable interactions for the charged fermions that take Yukawa only after $\mathrm{E}_6 \times S_3$ breaking.

We have to study the $\mathbf{E}_6{\times}\mathbf{S}_3$ **renormalizable** invariant Lagrangian that fits all data at the electroweak scale.

Thank you.

NEUTRINO DATA

The mixing angles and the Δm^2 measured are

	lower limit (3σ)	best value	upper limit (3σ)
$(\Delta m^2_{sol})_{LA}(10^{-5}\ eV^2)$	5.4	6.9	9.5
$(\Delta m^2_{atm})(10^{-3}\ eV^2)$	1.4	2.6	3.7
$\sin^2\theta_{12}$ (sol)	0.23	0.30	0.39
$\sin^2\theta_{23}$ (atm)	0.31	0.52	0.72
$\sin^2\theta_{13}$	0	0.006	0.054

Maltoni et al hep-ph/0309130

$$\sin^2\theta_{13} = 0, \;\; \sin^2\theta_{23} = 1/2, \;\; \sin^2\theta_{12} = 1/3$$

are in well agreement with data and the MNS is the following

$$U_{(HPS)} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad Harrison - Perkins - Scott$$

Harrison,Perkins e Scott, Phys.Lett.B 530(2002)

THE $S_3 \supset S_2$ NEUTRINO MODEL

$$L_{yuk} = g \sum_i \bar{\nu}_L^i \, \nu_R^i \, H + \lambda_1 \sum_i \bar{\nu}_R^{c\ i} \, \nu_R^i \, \phi^i + \lambda_2 \sum_{i,j} \bar{\nu}_R^{c\ i} \, \nu_R^j \, \psi + h.c.$$

$$M_\nu^D = gv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_\nu^R = \langle\psi\rangle \begin{pmatrix} \frac{\langle\phi^1\rangle}{\langle\psi\rangle} + 1 & 1 & 1 \\ 1 & \frac{\langle\phi^2\rangle}{\langle\psi\rangle} + 1 & 1 \\ 1 & 1 & \frac{\langle\phi^3\rangle}{\langle\psi\rangle} + 1 \end{pmatrix}$$

that can be diagonalized as follows

$$O^T(\theta_{sol}) \begin{pmatrix} \frac{\langle\phi^1\rangle}{\langle\psi\rangle} + 1 & 1 & 1 \\ 1 & \frac{\langle\phi^2\rangle}{\langle\psi\rangle} + 1 & 1 \\ 1 & 1 & \frac{\langle\phi^3\rangle}{\langle\psi\rangle} + 1 \end{pmatrix} O(\theta_{sol}) = \begin{pmatrix} m_{R1} & 0 & 0 \\ 0 & m_{R2} & 0 \\ 0 & 0 & m_{R3} \end{pmatrix}$$

$$\Rightarrow \quad \sqrt{2} \sin 2\,\theta_{sol} - \cos 2\,\theta_{sol} = \frac{m_{R_1} + m_{R_2} - 2\, m_{R_3}}{m_{R_2} - m_{R_1}} = R$$

where $m_{R_2} \sim \langle\psi\rangle$ S_3 singlet mass. Assuming $\langle\psi\rangle \gg \langle\phi_i\rangle$

$$R \to 1 \quad \Rightarrow \quad \sin\theta_{sol} = 1/\sqrt{3}$$

The $E_6 \times S_3$ full lagrangian

It is the following

$$L = L_\nu + L_{\text{charged fermions}}$$

$$\begin{aligned}
L_\nu \;=\;& 27^\alpha_i\ 27^\beta_i\ 351'_{\gamma\delta} + 27^\alpha_i\ 27^\beta_i\ 27^*_{k\,\alpha}\ 27^*_{k\,\beta} \\
& + 27^\alpha_i\ 27^\beta_j\ \left(351^k_{\gamma\alpha}\ 78^\gamma_\beta + 351^k_{\gamma\beta}\ 78^\gamma_\alpha\right) \\
& + 27^\alpha_i\ 27^\beta_i\ \left(351^i_{\gamma\alpha}\ 78^\gamma_\beta + 351^i_{\gamma\beta}\ 78^\gamma_\alpha\right)
\end{aligned}$$

$$\begin{aligned}
L_{\text{fc}} \;=\;& 27^\alpha_i\ 27^\beta_i\ 351'_{\gamma\delta}\, 2430^{i\ \ \gamma\delta}_{\alpha\beta} + \\
& + 27^\alpha_i\ 27^\beta_i\ 351'^{*\ \gamma\delta}\ 1728^\rho_{i\ \ \beta\delta}\, \varepsilon_{\alpha\gamma\rho} \\
& + 27^\alpha_i\ 27^\beta_j\ 351'^{*\ \gamma\delta}\ 1728^\rho_{i\ \ \beta\delta}\, \varepsilon_{\alpha\gamma\rho} + \\
& + 27^\alpha_i\ 27^\beta_i\ 27^\gamma_k\, \varepsilon_{\alpha\beta\gamma}
\end{aligned}$$

$\mathrm{E}_6 \supset \mathrm{SU}(2)\times\mathrm{SU}(6) \supset \mathrm{SU}(5)\times\mathrm{U}_{T_3}(1)\times\mathrm{U'}(1)$

$$Q' = -\frac{3}{4}Q_r + \frac{5}{4}Q_t, \quad Q_{T_3} = -\frac{1}{4}Q_r - \frac{1}{4}Q_t$$

$$27 = (2,\bar{6}) + (1,15) = R + T =$$

$$= \begin{pmatrix} \mathbf{v}^c_{R1} & D_1 & D_2 & D_3 & \bar{N}_L & \bar{E}_L \\ \mathbf{x}_L & d^c_{R1} & d^c_{R2} & d^c_{R3} & v_L & e_L \end{pmatrix}_{a\alpha} + \begin{pmatrix} 0 & \bar{D}_1 & \bar{D}_2 & \bar{D}_3 & -N_L & E_L \\ -\bar{D}_1 & 0 & u^c_{R3} & -u^c_{R2} & -d_{L1} & u_{L1} \\ -\bar{D}_2 & -u^c_{R3} & 0 & u^c_{R1} & -d_{L2} & u_{L2} \\ -\bar{D}_3 & u^c_{R2} & -u^c_{R1} & 0 & -d_{L3} & u_{L3} \\ N_L & d_{L1} & d_{L2} & d_{L3} & 0 & e^c_R \\ -E_L & -u_{L1} & -u_{L2} & -u_{L3} & -e^c_R & 0 \end{pmatrix}_{\alpha\beta}$$

$\mathrm{SU}_L(2)$ is contained in SU(6) and SU(6)⊃SU(5)×U'(1)

$$6 = \mathbf{1}(-5) + 5(1), \quad 15 = 5(-4) + 10(2)$$

$$\mathrm{SU}(2)\times\mathrm{SU}(6) \xrightarrow{\langle 78\rangle} \mathrm{U}(1)\times\mathrm{SU}(6) \xrightarrow{\langle 351\rangle} \mathrm{SU}(5)\times\mathrm{U}_x(1).$$