Predictions from non trivial quark-lepton complementarity

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Overwiew

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- Phenomenology and Non trivial quark-lepton complementarity

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- Prediction for θ_{13}^{PMNS}

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The clear non trivial structure fo V_M and gauge coupling unification allow us to obtain in a straightforward way constraints on the high energy spectrum too.

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We obtain this prediction by first showing that the matrix V_M, product of the CKM and PMNS mixing matrices, may have a zero (1,3) entry which is favored by experimental data.

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Hence models with bimaximal or tribimaximal forms of the correlation matrix V_M are quite possible. Any theoretical model with a vanishing (1,3) entry of V_M that is in agreement with quark data, solar, and atmospheric mixing angle leads to $\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ$. This value is consistent with the present 90% CL experimental upper limit.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ$(cont'd)

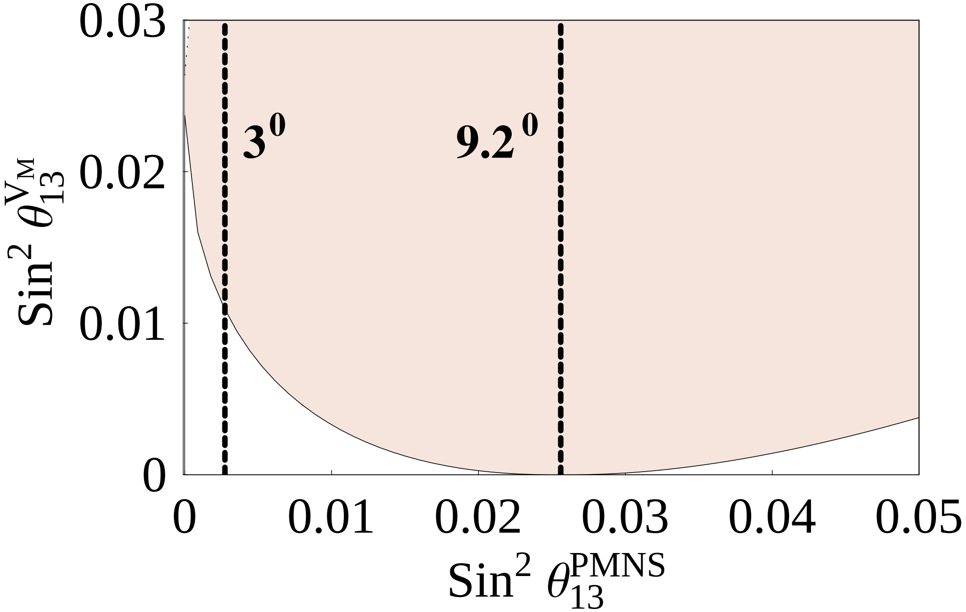

Figure 1: The minimum value allowed for $\sin^2\theta_{13}^{V_M}$ as a function of $\sin^2\theta_{13}^{PMNS}$. All the other CKM and PMNS mixing parameters are fixed at their best fit points.We also report the values of $\theta_{13}^{PMNS} = 3.0^\circ$ and 9.2°.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ$(cont'd)

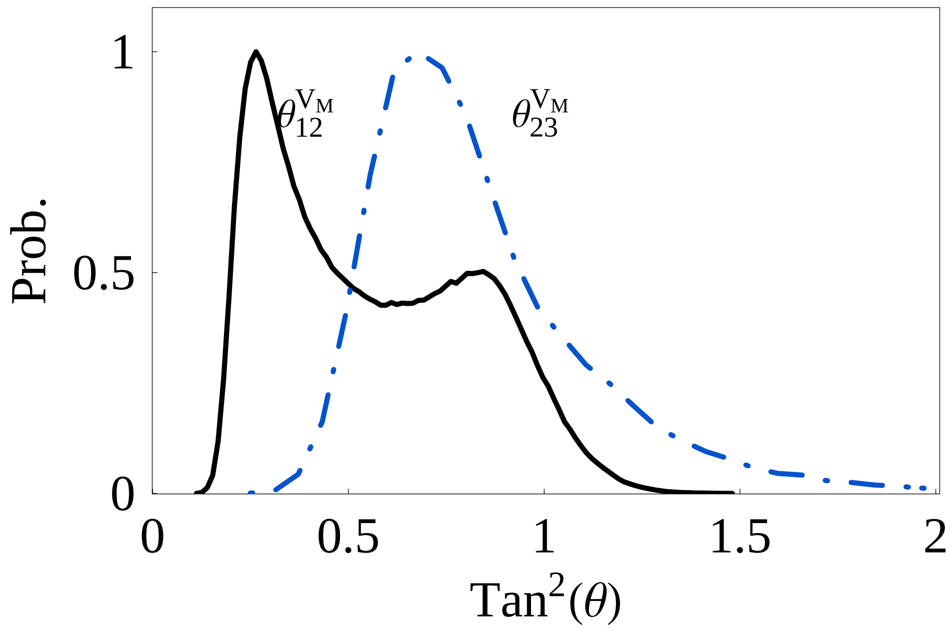

Figure 2: The distributions, normalized to one at the maximum, of $\tan^2\theta_{12}^{V_M}$ (continuos), and $\tan^2\theta_{23}^{V_M}$ (dot-dashed) obtained from the definition of the correlation mixing matrix V_M by using a Monte Carlo simulation of all the experimental data.

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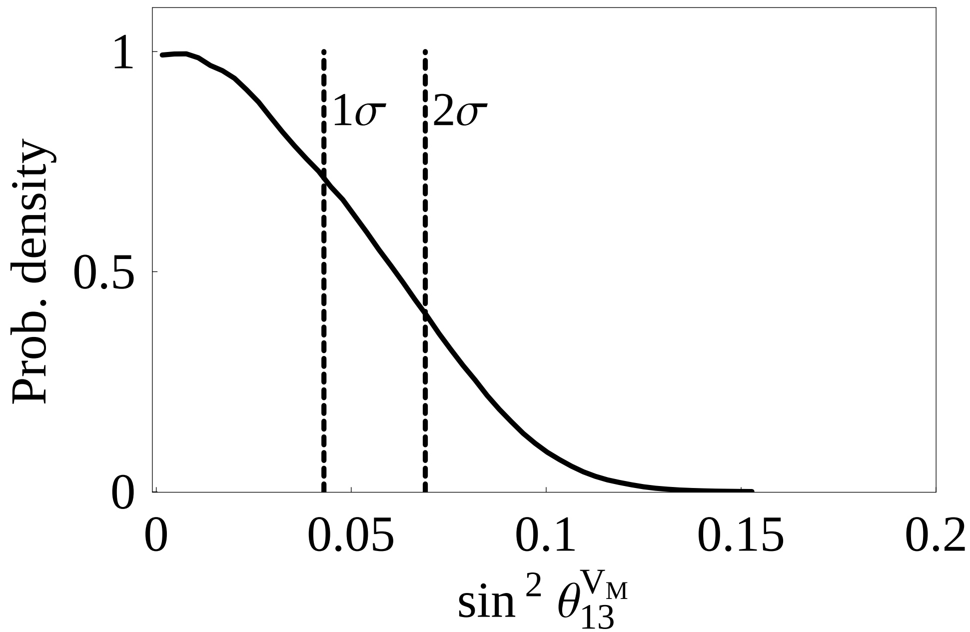

Figure 3: The distribution, normalized to one at the maximum, of $\sin^2\theta_{13}^{V_M}$ obtained from the definition of the correlation mixing matrix V_M by using a Monte Carlo simulation of all the experimental data. We also plot the 1σ and the 2σ lines.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ$(cont'd)

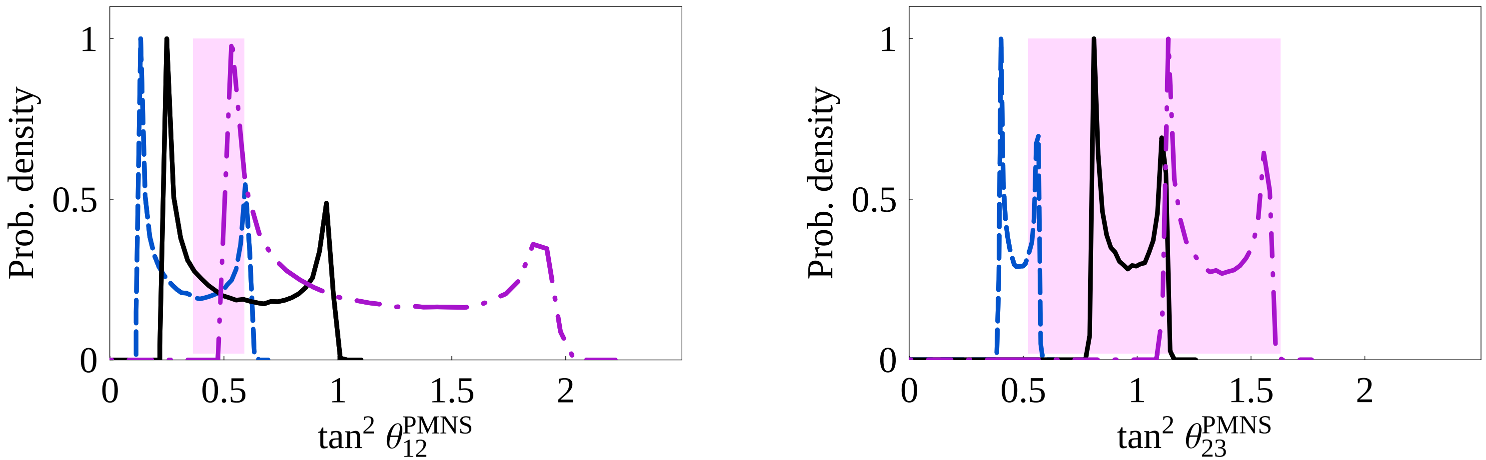

Figure 4: The distribution of $\tan^2\theta_{12}^{PMNS}$ (left), and $\tan^2\theta_{23}^{PMNS}$ (right) for the CKM experimental data and for values of the correlation matrix V_M respectively given by (left) $\tan^2\theta_{12}^{V_M} = 0.3$ (dashed), 0.5 (continuos), 1.0 (dot-dashed), $\tan^2\theta_{23}^{V_M} = 1.0$, and $\sin^2\theta_{13}^{V_M} = 0$; (right) $\tan^2\theta_{23}^{V_M} = 0.5$ (dashed), 1.0 (continuos), 1.4 (dot-dashed), $\tan^2\theta_{12}^{V_M} = 0.5$, $\sin^2\theta_{13}^{V_M} = 0$; The shaded areas represent the experimentally allowed regions for each case.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9^{+1}_{-2})^{\circ}$(cont'd)

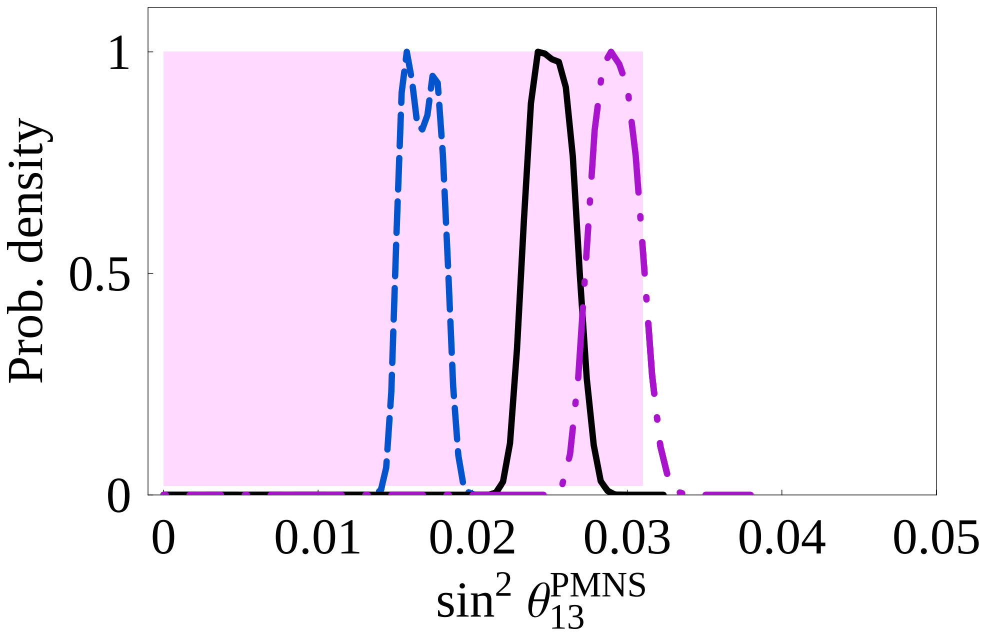

Figure 5: The distribution of $\sin^2\theta_{13}^{PMNS}$ for values of the correlation matrix V_M given by $\tan^2\theta_{12}^{V_M} = 0.5$, $\sin^2\theta_{13}^{V_M} = 0$, $\tan^2\theta_{23}^{V_M} = 0.5$ (dashed), 1.0 (continuos), 1.4 (dot-dashed). The shaded area represent the experimentally allowed region.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ$(cont'd)

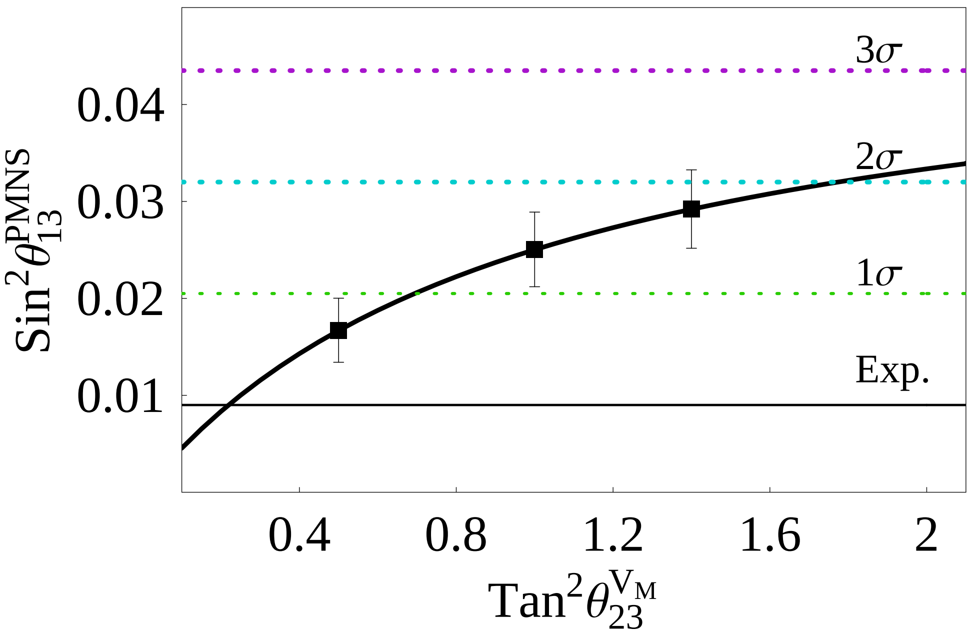

Figure 6: The allowed values for $\sin^2\theta_{13}^{PMNS}$ as a function of $\tan^2\theta_{23}^{V_M}$ under the assumption that $\sin^2\theta_{13}^{V_M} = 0$. We report the central and 3 σ values obtained from fig.5, and the approximate analytical dependence.We also plot the experimental value.

Neutrino CP violating parameters from non trivial quark-lepton complementarity

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- The Jarlskog invariant J that parametrizes the effects related to the Dirac phase

$$\begin{aligned} J &= Im\{U_{\nu_e\nu_1}U_{\nu_\mu\nu_2}U^*_{\nu_e\nu_2}U^*_{\nu_\mu\nu_1}\} \\ &= \frac{1}{8}\sin 2\theta_{12}\ \sin 2\theta_{23}\ \sin 2\theta_{13}\ \cos\theta_{13}\ \sin\phi\,. \end{aligned}$$

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&= \frac{1}{8}\sin 2\theta_{12}\ \sin 2\theta_{23}\ \sin 2\theta_{13}\ \cos\theta_{13}\ \sin\phi\,.
\end{aligned}
$$

- the two invariants S_1 and S_2 that parametrize the effects related to the Majorana phases

$$
\begin{aligned}
S_1 &= Im\{U_{\nu_e\nu_1} U^*_{\nu_e\nu_3}\} = \frac{1}{2}\cos\theta_{12}\sin 2\theta_{13}\sin(\phi+\phi_1) \\
S_2 &= Im\{U_{\nu_e\nu_2} U^*_{\nu_e\nu_3}\} = \frac{1}{2}\sin\theta_{12}\sin 2\theta_{13}\sin(\phi+\phi_2)
\end{aligned}
$$

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

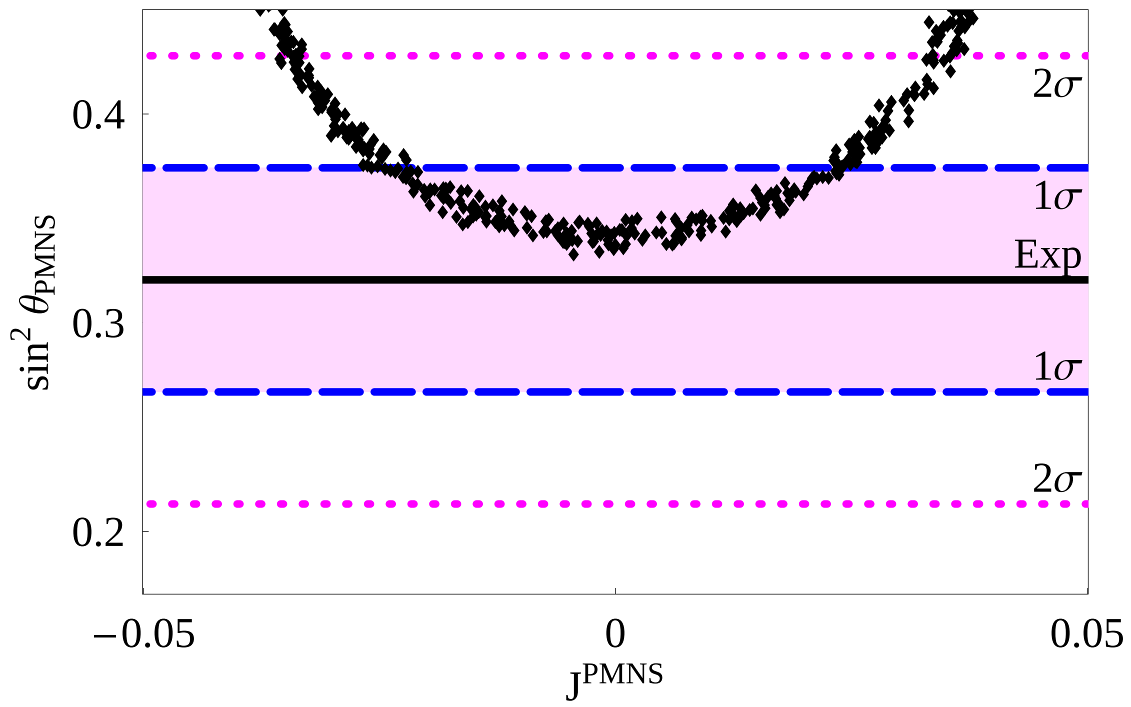

Figure 7: The correlation between the Dirac CP violating parameter J and $\sin^2\theta_{12}^{PMNS}$, for V_M bimaximal. We also plot the experimental central value, the 1σ, and the 2σ for $\sin^2\theta_{12}^{PMNS}$.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

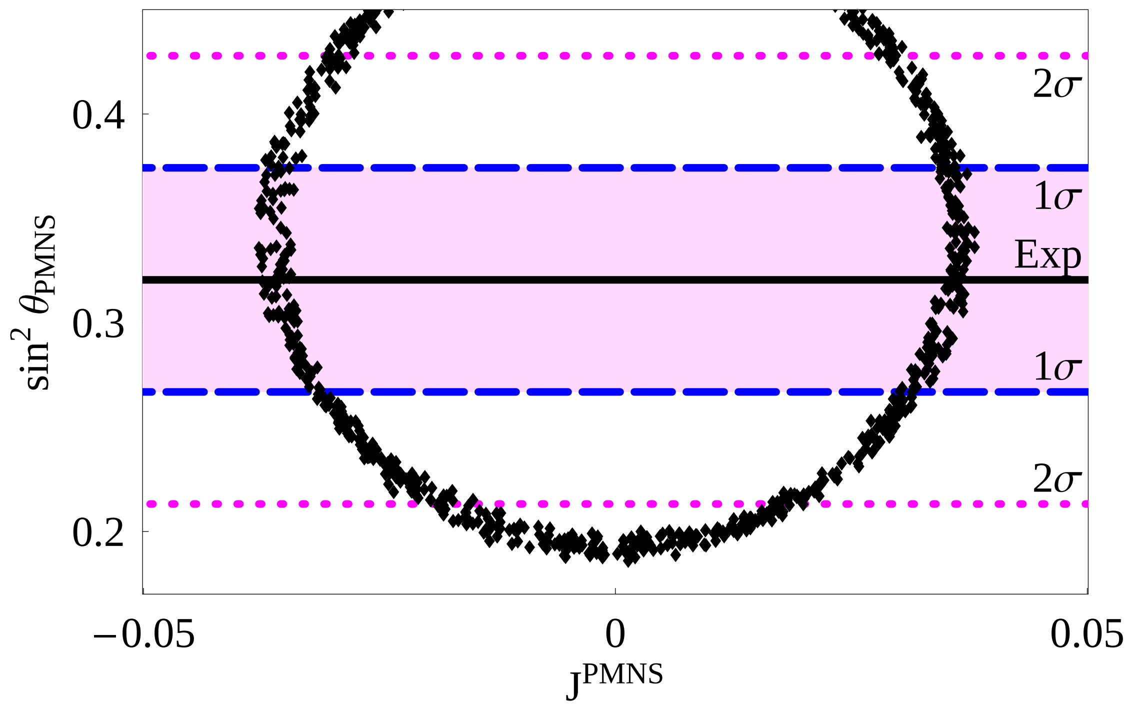

Figure 8: The correlation between the Dirac CP violating parameter J and $\sin^2\theta_{12}^{PMNS}$, for V_M tribimaximal. We also plot the experimental central value, the 1σ, and the 2σ for $\sin^2\theta_{12}^{PMNS}$.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

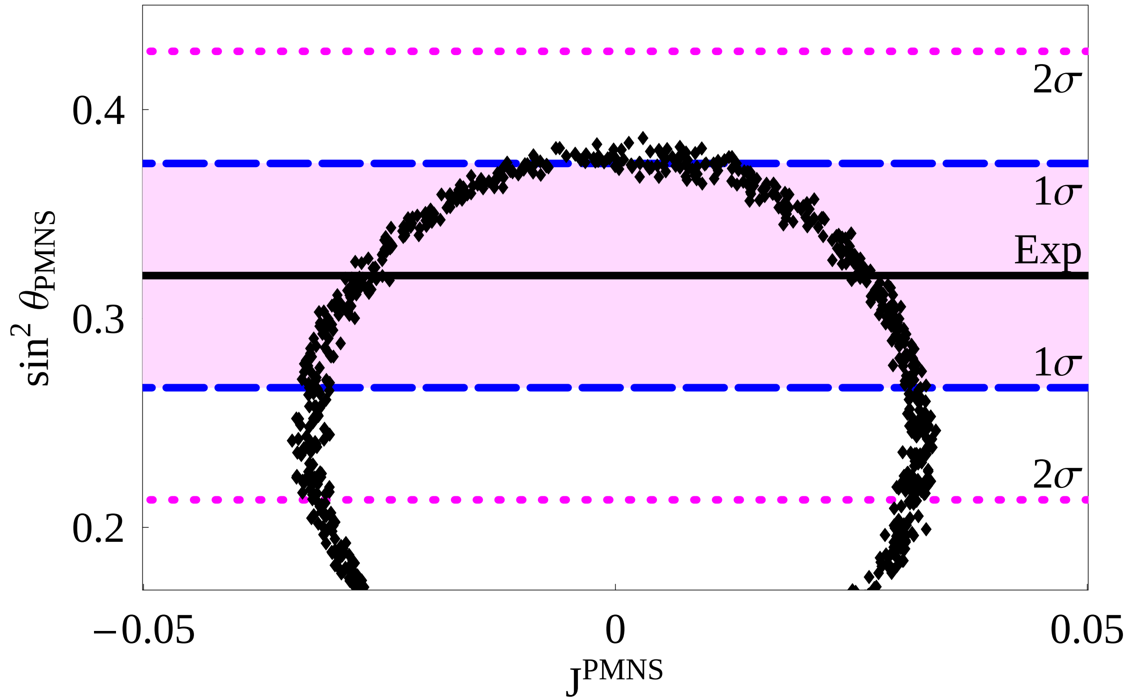

Figure 9: The correlation between the Dirac CP violating parameter J and $\sin^2\theta_{12}^{PMNS}$, for V_M with $\tan^2\theta_{12}^{V_M} = 0.3$. We fixed $\tan^2\theta_{23}^{V_M} = 1$ and $\sin^2\theta_{13}^{V_M} = 0$.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

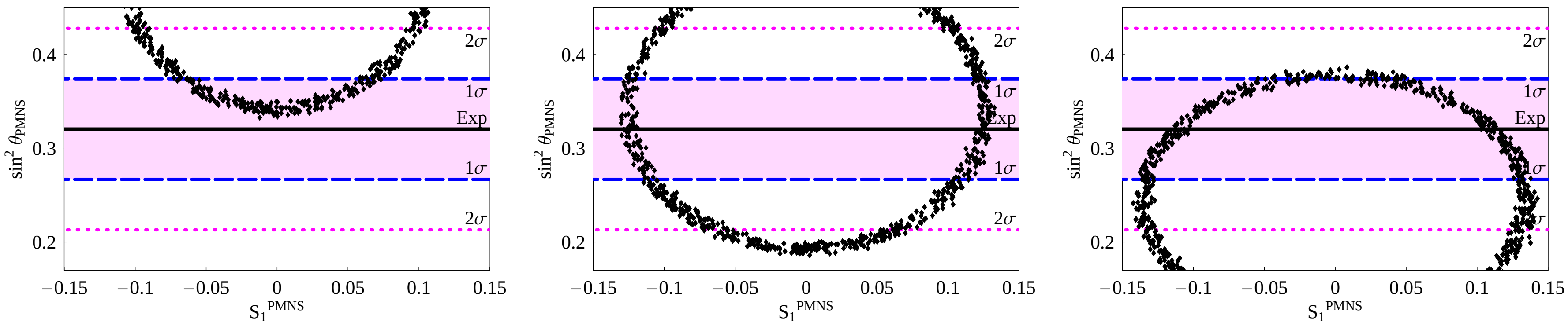

Figure 10: The correlation between the Majorana CP violating parameter S_1 and $\sin^2\theta_{12}^{PMNS}$, for V_M bimaximal, tribimaximal, and V_M with $\tan^2\theta_{12}^{V_M} = 0.3$. We fixed $\tan^2\theta_{23}^{V_M} = 1$ and $\sin^2\theta_{13}^{V_M} = 0$.

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$l_i \longrightarrow l_j \gamma$

$$BR(l_i \to l_j \gamma) \quad \propto \quad \frac{\Gamma(l_i \to e\nu\nu)}{\Gamma(l_i)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \times \times \left| \left(\tilde{M}_D L \tilde{M}_D^\dagger \right)_{ij} \right|^2$$

m_0 = scalar mass, A_0 = trilinear coupling,
$m_s^8 \approx 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$, $M_{1/2}$ = gaugino mass.

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The experimental limit for the branching ratio of $\mu \rightarrow e\gamma$ is 1.2×10^{-11} at 90% of confidence level and it could go down to 10^{-14} as proposed by MEG collaboration.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Leptogenesis

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The decay asymmetry can be written as

$$\epsilon_i \quad = \quad \frac{1}{8\pi v_u^2} \frac{1}{\left(\tilde{M}_D^\dagger \tilde{M}_D\right)_{ii}} \sum_{j \neq i} Im\left(\left(\tilde{M}_D^\dagger \tilde{M}_D\right)^2_{ji}\right) g(M_i^R, M_j^R)$$

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The $g(M_i^R, M_j^R)$ function is given by

$$g(x_i, x_j) = \left(\frac{2 x_i x_j}{x_i^2 - x_j^2} - \frac{x_j}{x_i} \log \frac{x_i^2 + x_j^2}{x_j^2}\right)$$

and M_i^R are the masses of the heavy neutrino.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$\mathcal{L} = \bar{\nu}_L Y_D \nu_R H + \nu_R^T C M_R \nu_R + \bar{l}_L Y_l l_R H + \bar{\nu}_L \not{W} l_L$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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We introduce the following definitions

$$l'_R = V_l^\dagger\, l_R \qquad \nu'_R = V_R^T\, \nu_R \qquad l'_L = U_l^\dagger\, l_L \qquad \nu'_L = U_l^\dagger\, \nu_L$$

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Consequently we have

$$l_R = V_l\, l'_R\,,\ \nu_R = V_R^*\, \nu'_R\,,\ l_L = U_l\, l'_L \quad \text{and}$$
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Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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In this primed base we get

$$\mathcal{L} \;=\; \bar{\nu}'_L U_l^\dagger M_D V_R^* \nu'_R + (\nu'_R)^T C M_R^\Delta \nu'_R + \bar{l}'_L M_l^\Delta l'_R + \bar{\nu}'_L \not{W}\, l'_L$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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and from $U_{CKM} = U_u^\dagger U_d$ we obtain

$$\tilde{M}_D = U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^\star$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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In see-saw of type I we have

$$
\begin{aligned}
m_{low}^\Delta &= U_{PMNS}^\dagger M_D \frac{1}{M_R} M_D^T U_{PMNS}^\star \\
&= U_{PMNS}^\dagger U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger V_0^\star M_D^\Delta U_{CKM}^\star U_{PMNS}^\star \\
V_M m_{low}^\Delta V_M^T &= M_D^\Delta V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star M_D^\Delta
\end{aligned}
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$$\begin{aligned} m_{low}^\Delta &= U_{PMNS}^\dagger M_D \frac{1}{M_R} M_D^T U_{PMNS}^\star \\ &= U_{PMNS}^\dagger U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger V_0^\star M_D^\Delta U_{CKM}^\star U_{PMNS}^\star \\ V_M m_{low}^\Delta V_M^T &= M_D^\Delta V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star M_D^\Delta \end{aligned}$$

We multiply on the left and on the right both sides by $1/M_D^\Delta$ and we get

$$V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star = \frac{1}{M_D^\Delta} V_M m_{low}^\Delta V_M^T \frac{1}{M_D^\Delta}$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

One can extract $V_0^\dagger V_R^\star$ by making the *square root* of the matrices

$$V_0^\dagger V_R^\star \sqrt{\frac{1}{M_R^\Delta}} \;=\; \frac{1}{M_D^\Delta} V_M \sqrt{m_{low}^\Delta} R^T$$

where $R^T R = \mathbf{1}$, and one obtains that $V_0^\dagger V_R^\star = \frac{1}{M_D^\Delta} V_M \sqrt{m_{low}^\Delta} R^T \sqrt{M_R^\Delta}$
Finally one concludes that

$$\begin{aligned} \tilde{M}_D &= U_l^\dagger U_0 M_D^\Delta \frac{1}{M_D^\Delta} V_M \sqrt{m_{low}^\Delta} R^T \sqrt{M_R^\Delta} \\ &= U_{PMNS} \sqrt{m_{low}^\Delta} R^T \sqrt{M_R^\Delta} \end{aligned}$$

Any information from V_M is hidden into the R matrix.

However $V_0^\dagger V_R^\star$ is unequivocally fixes and then the R matrix, once we know the eigenvalues of the Dirac neutrino mass matrix.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Determination **of** $V_0^\dagger V_R^\star$ **and** M_R^Δ**:** $V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star = \frac{1}{M_D^\Delta} V_M m_{low}^\Delta V_M^T \frac{1}{M_D^\Delta}$

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The experimental constraint says us that $(V_M)_{13}$ is zero:

$$V_M = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12}\cos\theta_{23} & \cos\theta_{12}\cos\theta_{23} & \sin\theta_{23} \\ \sin\theta_{12}\sin\theta_{23} & -\cos\theta_{12}\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

***Determination* of $V_0^\dagger V_R^\star$ and M_R^Δ:** $V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star = \frac{1}{M_D^\Delta} V_M m_{low}^\Delta V_M^T \frac{1}{M_D^\Delta}$

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$$V_M = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12}\cos\theta_{23} & \cos\theta_{12}\cos\theta_{23} & \sin\theta_{23} \\ \sin\theta_{12}\sin\theta_{23} & -\cos\theta_{12}\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

and the allowed ranges for $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ are

$$\tan^2\theta_{12}^{V_M} \in [0.3, 1.0] \qquad \text{and} \qquad \tan^2\theta_{23}^{V_M} \in [0.5, 1.4] \, .$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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Let us denote $m_{low}^\Delta = \{m_1, m_2, m_3\}$, and $M_D^\Delta = \{M_1, M_2, M_3\}$.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

We have three cases:

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

We have three cases:

1) hierarchical Dirac neutrino eigenvalues

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

We have three cases:

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2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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(in this case we have very hierarchical right-handed neutrino masses)

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(that implies right-handed neutrino close to be degenerate)

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Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Hierarchical Dirac neutrino eigenvalues

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Hierarchical Dirac neutrino eigenvalues

For any V_M, the heavy neutrino spectrum is hierarchical with ratios given by

$$M_1^R : M_2^R : M_3^R = M_1^2 : M_2^2 : M_3^2 = (\lambda^{4n})^2 : (\lambda^{2n})^2 : 1$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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Moreover the mixing matrix $V_0^\dagger V_R^\star$ is close to the identity.

$$V_0^\dagger V_R^\star \quad = \quad \begin{pmatrix} 1-\alpha^2\lambda^{2n}/2 & \alpha\lambda^n & \beta\lambda^{2n} \\ -\alpha\lambda^n & 1-(\alpha^2+\gamma^2)\lambda^{2n} & \gamma\lambda^n \\ (-\beta+\alpha\gamma)\lambda^{2n} & -\gamma\lambda^n & 1-\gamma^2\lambda^{2n}/2 \end{pmatrix} + O\left(\lambda^{3n}\right)$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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Moreover the mixing matrix $V_0^\dagger V_R^\star$ is close to the identity.

$$V_0^\dagger V_R^\star = \begin{pmatrix} 1-\alpha^2\lambda^{2n}/2 & \alpha\lambda^n & \beta\lambda^{2n} \\ -\alpha\lambda^n & 1-(\alpha^2+\gamma^2)\lambda^{2n} & \gamma\lambda^n \\ (-\beta+\alpha\gamma)\lambda^{2n} & -\gamma\lambda^n & 1-\gamma^2\lambda^{2n}/2 \end{pmatrix} + O\left(\lambda^{3n}\right)$$

In this case we have very hierarchical right-handed neutrino masses

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$
\begin{aligned}
m_\alpha &= m_1 \cos^2\theta_{12} + m_2 \sin^2\theta_{12} + O\left(\lambda^{2n}\right) && (1)\\
m_\beta &= \frac{m_1 m_2}{m_\alpha}\cos^2\theta_{23} + m_3 \sin^2\theta_{23} + O\left(\lambda^{2n}\right)\\
m_\gamma &= \frac{m_1 m_2 m_3}{m_\alpha m_\beta}\\
\alpha &= -\frac{(m_1 - m_2)}{2m_\alpha}\sin(2\theta_{12})\cos\theta_{23} + O\left(\lambda^{2n}\right)\\
\gamma &= \frac{m_1 m_2 - m_3 m_\alpha}{2m_\alpha m_\beta}\sin(2\theta_{23}) + O\left(\lambda^{2n}\right)\\
\beta &= \frac{(m_1 - m_2)}{2m_\alpha}\sin(2\theta_{12})\sin\theta_{23} + O\left(\lambda^{2n}\right) && (2)
\end{aligned}
$$

the number α, β, γ are of order 1 but the corresponding angles must be computed up to order λ^{6n} to obtain the right heavy neutrino masses. The parameters m_α, m_β, m_γ are of order of the common low energy neutrino mass m. Notice that the rotation angles $(1,2)$ and $(2,3)$ in $V_0^\dagger V_R^\star$ are of order λ^n while the $(1,3)$ angle is of order λ^{2n}.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$BR(\tau \to \mu\gamma) \quad \propto \quad \frac{\Gamma(\tau \to \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \hat{L}^2 \left(\frac{M_3}{m_t}\right)^4 \left| m_t^2 \log\frac{m_t^2}{m_u^2} A\lambda^2 \right|^2$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$BR(\tau \to \mu\gamma) \quad \propto \quad \frac{\Gamma(\tau \to \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \hat{L}^2 \left(\frac{M_3}{m_t}\right)^4 \left| m_t^2 \log\frac{m_t^2}{m_u^2} A\lambda^2 \right|^2$$

$BR(\mu \to e\gamma)$ is suppressed by a factor λ^6 with respect to $BR(\tau \to \mu\gamma)$, and by a factor λ^2 with respect to $BR(\tau \to e\gamma)$.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$BR(\tau \to \mu\gamma) \quad \propto \quad \frac{\Gamma(\tau \to \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \hat{L}^2$$
$$\left(\frac{M_3}{m_t}\right)^4 \left| m_t^2 \log\frac{m_t^2}{m_u^2} A\lambda^2 \right|^2$$

$BR(\mu \to e\gamma)$ is suppressed by a factor λ^6 with respect to $BR(\tau \to \mu\gamma)$, and by a factor λ^2 with respect to $BR(\tau \to e\gamma)$.
Our conclusions are equivalent to the one in literature for V_M bimaximal. However our discussion is more general, in fact we shown that these results do not depend on the form of the correlation matrix V_M.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Degenerate M_D and non degenerate m_{low}

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Degenerate M_D and non degenerate m_{low}

$$\frac{1}{M_R^\Delta} \simeq \begin{pmatrix} m_\alpha/M_3^2 & 0 & 0 \\ 0 & m_\beta/M_3^2 & 0 \\ 0 & 0 & m_\gamma/M_3^2 \end{pmatrix}$$

$$V_0^\dagger V_R^\star \simeq V_M \begin{pmatrix} 1-\alpha^2/2 & \alpha & \beta \\ -\alpha & 1-(\alpha^2+\gamma^2) & \gamma \\ (-\beta+\alpha\gamma) & -\gamma & 1-\gamma^2/2 \end{pmatrix} \equiv V_M V_\epsilon$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Degenerate M_D and non degenerate m_{low}

$$\frac{1}{M_R^{\Delta}} \simeq \begin{pmatrix} m_\alpha/M_3^2 & 0 & 0 \\ 0 & m_\beta/M_3^2 & 0 \\ 0 & 0 & m_\gamma/M_3^2 \end{pmatrix}$$

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In this case the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$
\begin{aligned}
m_\alpha &\simeq m_1 \left(1 - \frac{\delta M_1}{M_3}\left(1 + \frac{\cos(2\theta_{12})}{2}\right) + \frac{\delta M_2}{M_3}\left(-\frac{1-\cos(2\theta_{12})}{2} - \cos(2\theta_{23})\sin^2\theta_{12}\right)\right) \\
m_\beta &\simeq m_2 \left(1 - \frac{\delta M_1}{M_3}\left(1 - \frac{\cos(2\theta_{12})}{2}\right) + \frac{\delta M_2}{M_3}\left(-\frac{1-\cos(2\theta_{12})}{2} - \cos(2\theta_{23})\cos^2\theta_{12}\right)\right) \\
m_\gamma &\simeq m_3 \left(1 - \frac{\delta M_2}{M_3}(1+\cos(2\theta_{23}))\right) \\
\alpha &\simeq -\frac{m_1+m_2}{4(m_1-m_2)}\frac{2\delta M_1 - \delta M_2 - \delta M_2\cos(2\theta_{23})}{M_3}\sin(2\theta_{12}) \\
\gamma &\simeq \frac{m_2+m_3}{2(m_2-m_3)}\frac{\delta M_2}{M_3}\sin(2\theta_{23})\cos\theta_{12} \\
\beta &\simeq \frac{m_1+m_3}{2(m_1-m_3)}\frac{\delta M_2}{M_3}\sin(2\theta_{23})\sin\theta_{12} \qquad (3)
\end{aligned}
$$

the parameters m_α, m_β, m_γ are of order of the common low energy neutrino mass m.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$BR(\mu \to e\gamma) \quad \propto \quad \left| M_1 M_2 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \cos\alpha_{23} \sin\alpha_{12} \right|^2$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$BR(\mu \to e\gamma) \quad \propto \quad \left| M_1 M_2 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \cos\alpha_{23} \sin\alpha_{12} \right|^2$$

$$BR(\tau \to e\gamma) \quad \propto \quad \left| M_1 M_3 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \sin\alpha_{23} \sin\alpha_{12} \right|^2$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$
\begin{aligned}
BR(\mu \to e\gamma) &\propto \left| M_1 M_2 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \cos\alpha_{23} \sin\alpha_{12} \right|^2 \\
BR(\tau \to e\gamma) &\propto \left| M_1 M_3 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \sin\alpha_{23} \sin\alpha_{12} \right|^2 \\
BR(\tau \to \mu\gamma) &\propto \left| M_2 M_3 \cos\alpha_{23} \sin\alpha_{23} \left(\log \frac{m_\gamma}{m_\alpha} + \sin^2\alpha_{12} \log \frac{m_\beta}{m_\alpha} \right) \right|^2
\end{aligned}
$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

$$BR(\mu \to e\gamma) \quad \propto \quad \left| M_1 M_2 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \cos\alpha_{23} \sin\alpha_{12} \right|^2$$

$$BR(\tau \to e\gamma) \quad \propto \quad \left| M_1 M_3 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \sin\alpha_{23} \sin\alpha_{12} \right|^2$$

$$BR(\tau \to \mu\gamma) \quad \propto \quad \left| M_2 M_3 \cos\alpha_{23} \sin\alpha_{23} \left(\log \frac{m_\gamma}{m_\alpha} + \sin^2\alpha_{12} \log \frac{m_\beta}{m_\alpha} \right) \right|^2$$

The ratios among them become

$$\frac{BR(\mu \to e\gamma)}{BR(\tau \to e\gamma)} \quad \simeq \quad \tan^2\alpha_{23} \in [0.5, 1.4]$$

$$\frac{BR(\mu \to e\gamma)}{BR(\tau \to \mu\gamma)} \quad \simeq \quad \left| \frac{\cos\alpha_{12} \sin\alpha_{12}}{\sin\alpha_{23} \left((\log \frac{m_\gamma}{m_\alpha} / \log \frac{m_\beta}{m_\alpha}) + \sin^2\alpha_{12} \right)} \right|^2$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Degenerate M_D and Degenerate m_{low}

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Degenerate M_D and Degenerate m_{low}

$$M_1^R = \frac{m}{M^2}\left(1-\frac{\delta M_1}{M}\left(1+\sqrt{1-\frac{1}{3}\frac{\sqrt{\delta m^2_{sol}}/m}{\delta M_1/M}+\frac{\delta m^2_{sol}/m^2}{(\delta M_1/M)^2}}\right)\right)$$

$$M_2^R = \frac{m}{M^2}\left(1-2\frac{\delta M_2}{M}+\frac{\sqrt{\delta m^2_{atm}}}{m}\right)$$

$$M_3^R = \frac{m}{M^2}\left(1-\frac{\delta M_1}{M}\left(1-\sqrt{1-\frac{1}{3}\frac{\sqrt{\delta m^2_{sol}}/m}{\delta M_1/M}+\frac{\delta m^2_{sol}/m^2}{(\delta M_1/M)^2}}\right)\right)$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

Degenerate M_D and Degenerate m_{low}

$$
\begin{aligned}
M_1^R &= \frac{m}{M^2}\left(1-\frac{\delta M_1}{M}\left(1+\sqrt{1-\frac{1}{3}\frac{\sqrt{\delta m^2_{sol}}/m}{\delta M_1/M}+\frac{\delta m^2_{sol}/m^2}{(\delta M_1/M)^2}}\right)\right)\\
M_2^R &= \frac{m}{M^2}\left(1-2\frac{\delta M_2}{M}+\frac{\sqrt{\delta m^2_{atm}}}{m}\right)\\
M_3^R &= \frac{m}{M^2}\left(1-\frac{\delta M_1}{M}\left(1-\sqrt{1-\frac{1}{3}\frac{\sqrt{\delta m^2_{sol}}/m}{\delta M_1/M}+\frac{\delta m^2_{sol}/m^2}{(\delta M_1/M)^2}}\right)\right)
\end{aligned}
$$

The right-handed neutrino close to be degenerate and $V_0^\dagger V_R^\star$ is close to the identity

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

We get

$$
\begin{aligned}
BR(\mu \to e\gamma) &\propto \left|(M_2^2 - M_1^2)\right|^2 \lambda^2 \\
BR(\tau \to e\gamma) &\propto \left|((1-(\rho - i\eta))M_1^2 - M_2^2 + M_3^2(\rho - i\eta))\right|^2 (A\lambda^3)^2 \\
BR(\tau \to \mu\gamma) &\propto \left|(M_3^2 - M_2^2)\right|^2 (A\lambda^2)^2
\end{aligned}
$$

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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and $BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = 1 : \lambda^4 : \lambda^2$.

Predictions in SUSY from non trivial quark-lepton complementarity(cont'd)

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BR(\mu \to e\gamma) &\propto \left|(M_2^2 - M_1^2)\right|^2 \lambda^2 \\
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\end{aligned}$$

and $BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = 1 : \lambda^4 : \lambda^2$.
For example, if $M_2^2 - M_1^2 \simeq \lambda^4 M_3^2$ and M_i are of order m_t, we obtain

$$BR(\mu \to e\gamma) \propto \left(\frac{M_3}{m_t}\right)^4 \left|m_t^4 \lambda^5\right|^2 \quad , \qquad BR(\tau \to e\gamma) \propto \left(\frac{M_3}{m_t}\right)^4 \left|m_t^4 \lambda^7\right|^2$$

$$BR(\tau \to \mu\gamma) \quad \propto \quad \left(\frac{M_3}{m_t}\right)^4 \left|m_t^4 \lambda^6\right|^2$$

Conclusions

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- Phenomenology and Non trivial quark-lepton complementarity

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Conclusions(cont'd)

$l_i \to l_j \gamma$: we have three cases:

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1) hierarchical Dirac neutrino eigenvalues

(very hierarchical right-handed neutrino masses, and $V_0^\dagger V_R^\star \simeq I$)

$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = \lambda^6 : \lambda^4 : 1 \propto M_3^4 \lambda^4 \hat{L}.$

Conclusions(cont'd)

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$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = \lambda^6 : \lambda^4 : 1 \propto M_3^4 \lambda^4 \hat{L}$.

2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses (the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum, and $V_0^\dagger V_R^\star \simeq V_M$)
$BR(\mu \to e\gamma) \simeq BR(\tau \to e\gamma) \simeq BR(\tau \to \mu\gamma) \propto M_3^4$

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3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum (right-handed neutrino close to be degenerate, and $V_0^\dagger V_R^\star \simeq I$)
$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = 1 : \lambda^4 : \lambda^2 \propto M_3^4 \lambda^{10}$

Conclusions(cont'd)

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$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = 1 : \lambda^4 : \lambda^2 \propto M_3^4 \lambda^{10}$

Thank you!